

Sterile neutrinos in tau lepton decays

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Abstract

We study possible contributions of heavy sterile neutrinos ν_h to the decays $\tau^- \rightarrow e^\pm(\mu^\pm)\pi^\mp\pi^-$. From the experimental upper bounds on their rates we derive new constraints on the $\nu_h - \nu_\tau$ mixing in the mass region $140.5 \text{ MeV} \leq m_{\nu_h} \leq 1637 \text{ MeV}$. We discuss cosmological and astrophysical status of ν_h in this mass region and compare our constraints with those recently derived by the NOMAD collaboration.

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1 Introduction

Hypothetical sterile neutrinos ν_s , blind to the electroweak interactions, were invoked into particle physics in various contexts as possible means to resolve observed anomalies. The most prominent case deals with neutrino anomalies: solar neutrino deficit, atmospheric neutrino anomaly and results of the LSND neutrino experiment. Simultaneous explanation of all these three anomalies in terms of neutrino oscillations requires at least one sterile neutrino ν_s [1]. In this case ν_s together with $\nu_{e,\mu,\tau}$ allows one to introduce three independent mass square differences associated with these three anomalies. Although the recent Super Kamiokande global analysis [2] disfavours active-sterile neutrino oscillations as a dominant channel this hypothesis is not ruled out. Along this line one may reasonably admit the existence of more than one sterile neutrino, say, one per generation, as it is suggested by various extensions of the SM. In this situation among the neutrino mass eigenstates, which are superpositions of the active $\nu_{e,\mu,\tau}$ and sterile $\nu_{s(i)}$ weak eigenstates, one can encounter not only light neutrinos, but also heavy states ν_h . The question of viability of this scenario is the subject of experimental searches as well as cosmological and astrophysical constrains.

Recently some phenomenological, cosmological and astrophysical issues of the intermediate mass neutrinos ν_h in the MeV mass region have been addressed in the literature [3]. This was stimulated by the attempts of explanation of the KARMEN anomaly [4] in terms of a neutrino state with a mass of 33.9 MeV mixed with ν_τ [5]. Although recent data of this collaboration [6] have not confirmed this anomaly, the question of existence of heavy sterile neutrinos ν_h remains open.

These sterile neutrinos can be searched for as peaks in differential rates of various processes and by direct production of ν_h followed by their decays in the detector (for summary see Ref. [7] p. 361.). The ν_h can also give rise to significant enhancement of the total rate of certain processes if their masses happen to be located in an appropriate region [8]. This effect would be especially pronounced in reactions that are forbidden in the SM. The lepton number/flavor violating (LNV/LFV) processes belong to this category. Many of them are stringently restricted by experiment and allow one to derive stringent limits on the ν_h contribution.

In the present paper we study the ν_h contribution to the LFV and LNV τ -decays: $\tau^- \rightarrow e^-(\mu^-)\pi^+\pi^-$ and $\tau^- \rightarrow e^+(\mu^+)\pi^-\pi^-$. The first (LFV) process can receive contribution both from Dirac and Majorana neutrinos while to the second (LNV) process only Majorana neutrinos can contribute. These processes are capable to provide us with unique information on the $\nu_h - \nu_\tau$ mixing matrix

element $U_{\tau h}$. In the mass region $140.5\text{GeV} \leq m_h \leq 1637\text{MeV}$, which we are going to study, the ν_h contribution to the considered τ -decays gains resonant enhancement [8]. This effect makes discussed τ -decays very sensitive to presence of ν_h with the masses m_h in the resonant region. Under certain assumptions we extract from the experimental data [9] on these τ -decays new constraints on the $U_{\tau h}$ matrix element in the resonant region for both Majorana and Dirac heavy sterile neutrinos ν_h . Up to recently there were no credible laboratory limits on this quantity in the MeV mass region [7]. Only recent data of NOMAD collaboration [10] allowed establishing constraints on $U_{\tau h}$ in the MeV mass region. In that part of the resonant region, which overlaps with the region probed by NOMAD, our constraints are more stringent by around two orders of magnitude.

2 Neutrino mass matrix and interactions

Consider an extension of the SM with the three left-handed weak doublet neutrinos $\nu'_{Li} = (\nu'_{Le}, \nu'_{L\mu}, \nu'_{L\tau})$ and n species of the SM singlet right-handed neutrinos $\nu'_{Ri} = (\nu'_{R1}, \dots, \nu'_{Rn})$. The general mass term for this set of fields can be written as

$$\begin{aligned}
& - \frac{1}{2} \overline{\nu'} \mathcal{M}^{(\nu)} \nu'^c + \text{H.c.} = -\frac{1}{2} (\overline{\nu'_L}, \overline{\nu'^c_R}) \begin{pmatrix} \mathcal{M}_L & \mathcal{M}_D \\ \mathcal{M}_D^T & \mathcal{M}_R \end{pmatrix} \begin{pmatrix} \nu'^c_L \\ \nu'_R \end{pmatrix} + \text{H.c.} = \\
& - \frac{1}{2} \sum_{i=1}^{3+n} m_{\nu i} \overline{\nu'^c_i} \nu_i + \text{H.c.}
\end{aligned} \tag{1}$$

Here $\mathcal{M}_L, \mathcal{M}_R$ are 3×3 and $n \times n$ symmetric Majorana mass matrices, \mathcal{M}_D is $3 \times n$ Dirac type matrix. Rotating the neutrino mass matrix by the unitary transformation to the diagonal form

$$U^T \mathcal{M}^{(\nu)} U = \text{Diag}\{m_{\nu i}\} \tag{2}$$

we end up with $n + 3$ Majorana neutrinos $\nu_i = U_{ki}^* \nu'_k$ with the masses $m_{\nu i}$. In special cases there may appear among them pairs with masses degenerate in absolute values. Each of these pairs can be collected into a Dirac neutrino field. This situation corresponds to conservation of certain lepton numbers assigned to these Dirac fields.

The considered generic model must contain at least three observable light neutrinos while the other states may be of arbitrary mass. In particular, they may include hundred MeV neutrinos ν_h , which we are going to consider in the next section. Presence or absence of these neutrino states is a question for experimental searches.

In this scenario neutrino mass eigenstates have in general non-diagonal LFV couplings to the Z-boson

$$Z^\mu \sum_{\alpha=e,\mu,\tau} \bar{\nu}'_\alpha \gamma_\mu P_L \nu'_\alpha = Z^\mu \sum_{\alpha=e,\mu,\tau} \sum_{i,j=1}^{n+3} U_{\alpha i} U_{\alpha j}^* \bar{\nu}_j \gamma_\mu P_L \nu_i \equiv \sum_{i,j=1}^{n+3} \eta_{ij} \bar{\nu}_j \gamma_\mu P_L \nu_i, \quad (3)$$

where the last two expressions are written in the mass eigenstate basis. For the case of only three massive neutrinos one has $\mathcal{P}_{mn} = \delta_{mn}$ as a consequence of unitarity of $U_{\alpha n}$. In general \mathcal{P}_{mn} is not a diagonal matrix and flavor changing neutral currents in the neutrino sector become possible at tree level.

As is known the invisible Z-boson width Γ_{inv}^Z is an efficient counter of the number of neutrinos. In the introduced scenario, despite possible presence of an arbitrary number of light neutrinos, the predicted value of the Γ_{inv}^Z is always consistent with its experimental value. Indeed, if all the neutrinos are light with masses $m_{\nu i} \ll M_Z$ their contribution to the invisible Z-boson width can be written as

$$\Gamma_{inv}^Z = \sum_{i,j=1}^{n+3} |\eta_{ij}|^2 \Gamma_\nu^{SM} = \Gamma_\nu^{SM} \sum_{\alpha,\beta=e,\mu,\tau} \delta_{\alpha\beta} \delta_{\alpha\beta} = 3\Gamma_\nu^{SM}. \quad (4)$$

With the SM prediction for the partial Z-decay width to a pair of light neutrinos $\Gamma_\nu^{SM} = 167.24 \pm 0.08$ MeV this formula always reproduces the experimental value of $\Gamma_{inv}^Z = 498.8 \pm 1.5$ MeV. The chain of equalities in Eq. (4) follows again from the unitarity of $U_{\alpha n}$. Thus, independently of the number of light neutrinos with masses $m_\nu \ll M_Z$ the factor 3 in the last step counts the number of weak doublet neutrinos. This conclusion is changed in the presence of heavy neutrinos N with masses $M_N > M_Z/2$ which do not contribute to Γ_{inv} . In this case the unitarity condition is no longer valid and the factor 3 is changed to a smaller value.

Having these arguments in mind we introduce in the next section neutrino states ν_h with masses in the hundred MeV region. These states can be composed of sterile and active neutrino flavors as described in the present section.

3 $\tau^- \rightarrow e^\pm(\mu^\pm)\pi^\mp\pi^-$ decay rates

Neutrinos contribute to the LNV τ -decay $\tau^- \rightarrow l^+\pi^-\pi^-$ according to the lowest order diagrams shown in Fig.1. The ν -contribution to the LFV τ -decay $\tau^- \rightarrow l^-\pi^+\pi^-$ is determined by the tree-level diagram similar to the diagram in Fig. 1(a). The loop-diagram analogous to the diagram in Fig. 1(b) is absent in the

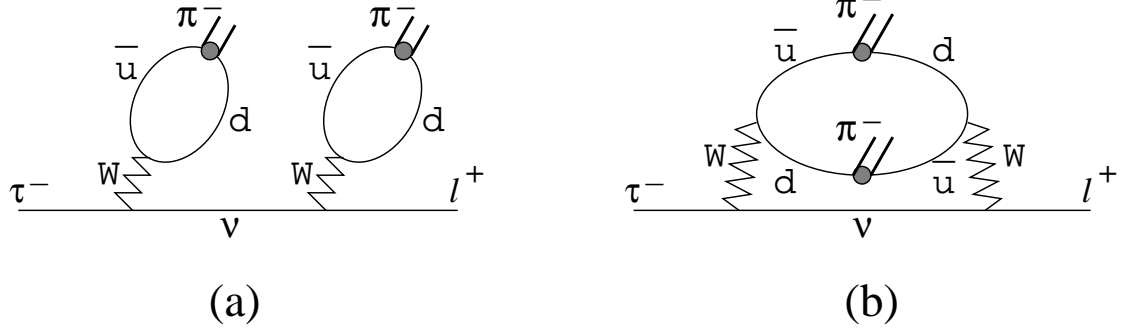


Figure 1: The lowest order diagrams contributing to $\tau^- \rightarrow l^+ \pi^+ \pi^-$ decay.

latter case. The diagram in Fig. 1(b) requires knowledge of the π -meson wave function at long distances. As yet it is poorly known and would introduce into the calculations uncontrollable uncertainties. In the present paper we concentrate on the resonant mass domain determined in (16), where the diagram in Fig. 1(a) absolutely dominates over that in Fig. 1(b). For this reason we neglect this diagram latter on. The hadronic structure parameters necessary for calculation of the tree-level contribution Fig. 1(a) involves only one parameter of hadronic structure, the pion decay constant f_π , accurately known from experiment. For the $\tau^- \rightarrow l^\pm \pi^\mp \pi^-$ decay with $l = e, \mu$ we derive the following decay rate formula

$$\Gamma(\tau^- \rightarrow l^+ \pi^- \pi^-) = c \int_{s_l^-}^{s^+} ds \left| \sum_k \frac{U_{lk} U_{\tau k} m_{\nu k}}{s - m_{\nu k}^2} \right|^2 G^l\left(\frac{s}{m_\tau^2}\right) + 2 \frac{c}{m_\tau^2} \text{Re} \left[\int_{s_l^-}^{s^+} ds \left(\sum_k \frac{U_{lk} U_{\tau k} m_{\nu k}}{s - m_{\nu k}^2} \right) \int_{v_l^-}^{v_l^+} dv \left(\sum_n \frac{U_{ln} U_{\tau n} m_{\nu n}}{v - m_{\nu n}^2} \right)^* H^l\left(\frac{s}{m_\tau^2}, \frac{v}{m_\tau^2}\right) \right], \quad (5)$$

$$\Gamma(\tau^- \rightarrow l^- \pi^+ \pi^-) = c \int_{s_l^-}^{s^+} ds \left| \sum_k \frac{U_{lk}^* U_{\tau k}}{s - m_{\nu k}^2} \right|^2 s G^l\left(\frac{s}{m_\tau^2}\right). \quad (6)$$

The unitary mixing matrix U_{ij} relates $\nu'_i = U_{ij} \nu_h$ weak ν' and mass ν neutrino eigenstates. The numerical constant is $c = (G_F^4/32)(\pi)^{-3} f_\pi^4 m_\tau^5 |V_{ud}|^4$, where $f_\pi =$

93MeV and $m_\tau = 1777\text{MeV}$ is the τ -lepton mass. We introduced the functions

$$\begin{aligned} G^l(z) &= \phi^l(z)z^{-2} \times [(z - x_l^2)^2 - x_\pi^2(z + x_l^2)][(z - 1)^2 - x_\pi^2(z + 1)], \\ H^l(z_1, z_2) &= (1 + x_l^2 - z_1 - z_2)(x_l^2 - z_1 z_2) + \\ &+ x_\pi^2 \left[(1 + x_l^2)(z_1 + z_2 - x_\pi^2) - 2(z_1 z_2 + x_l^2) \right], \end{aligned} \quad (7)$$

where $x_i = m_i/m_\tau$, $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$, $\phi^l(z) = \lambda^{1/2}(1, z, x_\pi^2)\lambda^{1/2}(z, x_\pi^2, x_l^2)$. The integration limits in Eqs. (5), (6) are

$$\begin{aligned} s_l^- &= m_\tau^2(x_\pi + x_l)^2, \quad s^+ = m_\tau^2(1 - x_\pi)^2, \\ v_l^\pm &= \frac{m_\tau^2}{2y} \left[(x_\pi^2 - x_l^2)(x_\pi^2 - 1) + y(1 + x_l^2 + 2x_\pi^2) - y^2 \pm \phi^l(y) \right] \end{aligned} \quad (8)$$

with $y = s/m_\tau^2$.

Assuming that there exist only light neutrinos with masses $m_{\nu i} \ll m_\tau$ we can rewrite Eqs. (5), (6) approximately in the form

$$\Gamma(\tau^- \rightarrow l^+ \pi^- \pi^-) = c \frac{|\langle m_\nu \rangle_{l\tau}|^2}{m_\tau^2} \mathcal{A}_+^{(l)}, \quad \Gamma(\tau^- \rightarrow l^- \pi^+ \pi^-) = c \frac{|\langle m_\nu^2 \rangle_{l\tau}|^2}{m_\tau^4} \mathcal{A}_-^{(l)}, \quad (9)$$

where

$$\begin{aligned} \mathcal{A}_+^{(l)} &= m_\tau^2 \int_{s_l^-}^{s^+} \frac{ds}{s^2} G^l\left(\frac{s}{m_\tau^2}\right) + 2 \int_{s_l^-}^{s^+} \frac{ds}{s} \int_{v_l^-}^{v_l^+} \frac{dv}{v} H^l\left(\frac{s}{m_\tau^2}, \frac{v}{m_\tau^2}\right), \\ \mathcal{A}_-^{(l)} &= m_\tau^4 \int_{s_l^-}^{s^+} \frac{ds}{s^3} G^l\left(\frac{s}{m_\tau^2}\right) \end{aligned} \quad (10)$$

and

$$\langle m_\nu \rangle_{l\tau} = \sum_k U_{lk} U_{\tau k} m_{\nu k}, \quad \langle m_\nu^2 \rangle_{l\tau} = \sum_k U_{lk}^* U_{\tau k} m_{\nu k}^2. \quad (11)$$

From atmospheric and solar neutrino oscillation data, combined with the tritium beta decay endpoint, one can derive upper bounds on masses of all the three neutrinos [11] $m_{e,\mu,\tau} \leq 3 \text{ eV}$. Then we have conservatively [12]

$$|\langle m_\nu \rangle_{\tau l}| < 9 \text{ eV}, \quad |\langle m_\nu^2 \rangle_{\tau l}| < 27 \text{ eV}^2. \quad (12)$$

With these upper bounds we obtain from Eqs. (9) a rough estimate

$$R_{e^+(\mu^+)} \equiv \frac{\Gamma(\tau^- \rightarrow e^+(\mu^+)\pi^-\pi^-)}{\Gamma(\tau^- \rightarrow All)} \lesssim 3.9(2.6) \times 10^{-31}, \quad (13)$$

$$R_{e^-(\mu^-)} \equiv \frac{\Gamma(\tau^- \rightarrow e^-(\mu^-)\pi^+\pi^-)}{\Gamma(\tau^- \rightarrow All)} \lesssim 3.0(1.2) \times 10^{-47}. \quad (14)$$

This is to be compared with the present experimental bounds on these branching ratios [7]

$$\mathcal{R}_{e^+(\mu^+)}^{exp} \leq 1.9(3.4) \times 10^{-6}, \quad \mathcal{R}_{e^-(\mu^-)}^{exp} \leq 2.2(8.2) \times 10^{-6}, \quad (15)$$

The comparison clearly shows that the light neutrino contributions to the processes $\tau^- \rightarrow l^\pm \pi^\mp \pi^-$ are far from being detected in the near future. On the other hand experimental observation of $\tau^- \rightarrow l^\pm \pi^\mp \pi^-$ decays at larger rates would indicate some new physics beyond the SM, or the presence of an extra neutrino state ν_h with the mass in the hundred MeV domain.

Assume that there exist neutrinos ν_h with masses m_h in the interval

$$\sqrt{s_e^-} \approx 140.5 \text{ MeV} \leq m_h \leq \sqrt{s^+} \approx 1637 \text{ MeV}. \quad (16)$$

These neutrinos would give resonant contributions to the processes $\tau^- \rightarrow l^\pm \pi^\mp \pi^-$ since the first term in Eqs. (5) and the expression in Eq. (6) have non-integrable singularities at $s = m_h^2$. Apparently, in the resonant region (16) one has to take into account the finite width Γ_{ν_h} of the neutrino ν_h . This is accomplished by the substitution $m_h \rightarrow m_h - (i/2)\Gamma_{\nu_h}$ in Eqs. (5), (6). As we will show in the next section the total decay width of the neutrino is small $\Gamma_{\nu_h} \ll m_{\nu_h}$. Therefore in the resonant domain (16) the neutrino propagator in Eqs. (5), (6) has a very sharp maximum at $s = m_h^2$. The second term in Eq. (5), being finite in the limit $\Gamma_{\nu_h} = 0$, can be neglected in this case. This allows us to rewrite Eqs. (5), (6) in the resonant mass region with good precision in a simple form

$$\Gamma^{res}(\tau^- \rightarrow l^\pm \pi^\mp \pi^-) \approx c\pi G^l(z_0) \frac{m_h |U_{\tau h}|^2 |U_{lh}|^2}{\Gamma_{\nu_h}}, \quad (17)$$

with $z_0 = (m_h/m_\tau)^2$.

4 Heavy sterile neutrino decays

Heavy sterile neutrinos ν_h can decay in both charged (CC) and neutral (NC) current channels $\nu_h \rightarrow l_1 l_2 \nu_2; l M(q_1 \bar{q}_2)$ and $\nu_h \rightarrow \nu_i l \bar{l}; \nu_i M(q \bar{q})$, where $l = e, \mu$

and M are mesons specified below. These decays are induced by the Lagrangian terms

$$\mathcal{L} = \frac{g_2}{\sqrt{2}} U_{li} \bar{l} \gamma^\mu P_L \nu_i W_\mu^- + \frac{g_2}{2 \cos \theta_W} U_{\alpha j} U_{\alpha i}^* \bar{\nu}_i \gamma^\mu P_L \nu_j Z_\mu, \quad (18)$$

where $l = e, \mu, \tau$ and $U_{\alpha\beta}$ is the neutrino mixing matrix defined in Eq. (2). Despite the fact that the NC term is of second order in the mixing matrix the NC decay channels of the heavy sterile neutrinos ν_h , as will be seen, are as important as the CC one.

Calculating the total width Γ_{ν_h} we divide the interval (16) into two parts

$$\begin{aligned} \sqrt{s_e^-} \approx 140.5 \text{ MeV} \leq m_h \leq m_{\eta'} = 958 \text{ MeV} & \quad \text{domain (I),} \\ m_{\eta'} = 958 \text{ MeV} < m_h \leq \sqrt{s^+} \approx 1637 \text{ MeV} & \quad \text{domain (II),} \end{aligned} \quad (19)$$

where $m_{\eta'} = 958 \text{ MeV}$ is the mass of the isoscalar pseudo-scalar meson $\eta'(958)$.

In the domain (I) we obtain Γ_{ν_h} directly, by calculating all the ν_h decay channels to the leptons l and mesons M one by one, using the following meson matrix elements

$$\begin{aligned} \langle \pi^-(p) | \bar{d} \gamma_\mu \gamma_5 u | 0 \rangle &= i \sqrt{2} f_\pi p_\mu, \quad \langle K^-(p) | \bar{s} \gamma_\mu \gamma_5 u | 0 \rangle = -i \sqrt{2} f_K p_\mu, \\ \langle \rho^-(p) | \bar{d} \gamma_\mu \gamma_5 u | 0 \rangle &= \sqrt{2} m_\rho f_\rho p_\mu \epsilon_\mu^*, \quad \langle K^{*-}(p) | \bar{s} \gamma_\mu \gamma_5 u | 0 \rangle = m_{K^*} f_{K^*} p_\mu \epsilon_\mu^*, \\ \langle \pi^0(p) | \bar{u} \gamma_\mu \gamma_5 u | 0 \rangle &= -\langle \pi^0(p) | \bar{d} \gamma_\mu \gamma_5 d | 0 \rangle = i f_\pi p_\mu, \\ \langle \eta(p) | \bar{u} \gamma_\mu \gamma_5 u | 0 \rangle &= \langle \eta(p) | \bar{d} \gamma_\mu \gamma_5 d | 0 \rangle = -\frac{1}{2} \langle \eta(p) | \bar{s} \gamma_\mu \gamma_5 s | 0 \rangle = i \frac{f_\pi}{\sqrt{3}} p_\mu, \\ \langle \rho^0(p) | \bar{u} \gamma_\mu u | 0 \rangle &= -\langle \rho^0(p) | \bar{d} \gamma_\mu d | 0 \rangle = m_\rho f_\rho \epsilon_\mu^*, \\ \langle \omega(p) | \bar{u} \gamma_\mu u | 0 \rangle &= \langle \omega(p) | \bar{d} \gamma_\mu d | 0 \rangle = m_\omega f_\omega \epsilon_\mu^*, \end{aligned} \quad (20)$$

where ϵ_μ is a vector meson polarization 4-vector; meson decay constants and masses are $f_\pi = 93 \text{ MeV}$, $f_K = 113 \text{ MeV}$, $f_\rho = 153 \text{ MeV}$, $f_{K^*} = 224 \text{ MeV}$, $f_\omega = 138 \text{ MeV}$, $m_\pi = 139.6 \text{ MeV}$, $m_K = 494 \text{ MeV}$, $m_\eta = 547 \text{ MeV}$, $m_\rho = 770 \text{ MeV}$, $m_\omega = 782 \text{ MeV}$, $m_{K^*} = 892 \text{ MeV}$.

This channel-by-channel approach can not be extended beyond the threshold of $\eta'(985)$ meson production since its decay constant $f_{\eta'}$ is unknown. Above the $\eta'(985)$ threshold, in the domain (II), we approximate the decay rate of $\nu_h \rightarrow l(\nu)M$ by the decay rate of the inclusive process $\nu_h \rightarrow l(\nu)q_1\bar{q}_2$ without perturbative and nonperturbative QCD corrections. This leading-order approximation should work for $m_{\nu_h} \gg \Lambda \approx 200 \text{ MeV}$. At lower masses a more viable

approach would be to relate the semileptonic ν_h decay rate by the dispersion relations to the imaginary parts of the W and Z self-energies $\Pi(s)$ in analogy to the approach applied in the literature for the $\tau \rightarrow \nu +$ hadrons inclusive decay [13]. However for our rough estimations we do not need this more sophisticated treatment and will use the above mentioned leading-order approximation.

The decay channels of the Dirac neutrino ν_h in the domain (I) are

$$(CC) : \quad \nu_h \longrightarrow \begin{cases} e^- e^+ \nu_e, & e^- \mu^+ \nu_\mu, & \mu^- e^+ \nu_e, & \mu^- \mu^+ \nu_\mu, \\ e^- \pi^+, & \mu^- \pi^+, & e^- K^+, & \mu^- K^+, \\ e^- \rho^+, & \mu^- \rho^+, & e^- K^{*+}, & \mu^- K^{*+}. \end{cases} \quad (21)$$

$$(NC) : \quad \nu_h \longrightarrow \begin{cases} \nu_i e^- e^+, & \nu_i \mu^+ \mu^-, & \nu_i \pi^0, \\ \nu_i \eta, & \nu_i \rho^0, & \nu_i \omega, \end{cases} \quad (22)$$

where $\nu_i = \nu_1, \nu_2, \nu_3$ are the three conventional light neutrino mass eigenstates. Latter on we neglect their masses since $m_{\nu i} \ll m_h$.

Partial decay widths for these channels are readily calculated using the hadronic matrix elements in Eq. (20). The results can be summarized as

$$\begin{aligned} \Gamma(\nu_h \rightarrow l_1 l_2 \nu) &= |U_{l_1 h}|^2 \frac{G_F^2}{192\pi^3} m_j^5 H(y_{l_1}, y_{l_2}) \equiv |U_{l_1 h}|^2 \Gamma_3^{(l_1 l_2)}, \\ \Gamma(\nu_h \rightarrow l P) &= |U_{lh}|^2 \frac{G_F^2}{4\pi} f_P^2 m_j^3 F_P(y_l, y_P) \equiv |U_{lj}|^2 \Gamma_2^{(lP)}, \\ \Gamma(\nu_h \rightarrow l V) &= |U_{lh}|^2 \alpha_V \frac{G_F^2}{8\pi} f_V^2 m_h^3 F_V(y_l, y_V) \equiv |U_{lj}|^2 \Gamma_2^{(lV)}, \\ \Gamma(\nu_h \rightarrow \nu_i l \bar{l}) &= |\zeta_i|^2 \frac{m_h^5 G_F^2}{192\pi^3} [a H_1^{(l)} + b H_2^{(l)}] \equiv |\zeta_i|^2 \Gamma_3^{(l)}, \\ \Gamma(\nu_h \rightarrow \nu_i P^0) &= |\zeta_i|^2 \frac{m_h^3 G_F^2}{64\pi} f_\pi^2 \alpha^{P^0} (1 - y_{P^0}^2)^2 \equiv |\zeta_i|^2 \Gamma_2^{(\nu P^0)}, \\ \Gamma(\nu_h \rightarrow \nu_i V^0) &= |\zeta_i|^2 \frac{m_h^3 G_F^2}{16\pi} f_{V^0}^2 (1 - y_{V^0}^2)^2 (1 + 2y_{V^0}^2) \equiv |\zeta_i|^2 \Gamma_2^{(\nu V^0)}, \end{aligned} \quad (23)$$

where

$$\zeta_i = \sum_{\alpha=e,\mu,\tau} U_{\alpha h} U_{\alpha i}^* \quad (24)$$

In Eq. (23) we denoted $P = \{\pi^+, K^+\}$, $V = \{\rho^+, K^{*+}\}$, $P^0 = \{\pi^0, \eta\}$, $V^0 = \{\rho^0, \omega\}$, $y_i = m_i/m_h$, $\alpha^{\pi^0} = 1$, $\alpha^\eta = 1/3$, $a = 2 \sin^4 \theta_W - \sin^2 \theta_W + 1/4$, $b =$

$\sin^2 \theta_W (\sin^2 \theta_W - 1/2)$. The kinematical functions are

$$\begin{aligned}
H_P(x, y) &= 12 \int_{z_1}^{z_2} \frac{dz}{z} (z - y^2)(1 + x^2 - z) \lambda^{1/2}(1, z, x^2) \lambda^{1/2}(0, y^2, z), \\
H_1^{(l)} &= 12 \int_w^1 \frac{dz}{\sqrt{z}} (z - 2y_l^2)(1 - z)^2 \sqrt{z - 4y_l^2}, \\
H_2^{(l)} &= 24y_l^2 \int_w^1 \frac{dz}{\sqrt{z}} (1 - z)^2 \sqrt{z - 4y_l^2}, \\
F_P(x, y) &= \lambda^{1/2}(1, x^2, y^2) [(1 + x^2)(1 + x^2 - y^2) - 4x^2], \\
F_V(x, y) &= \lambda^{1/2}(1, x^2, y^2) [(1 - x^2)^2 + (1 + x^2)y^2 - 2y^4],
\end{aligned}$$

where the integration limits are $z_1 = y^2$, $z_2 = (1 - x)^2$ and $w = 4y_l^2$.

In the mass domain (II) of Eq. (19) we approximate the CC and NC decay rates by the rates of the inclusive reactions

$$(CC) : \nu_h \longrightarrow l^- q_1 \bar{q}_2, \quad (NC) : \nu_h \longrightarrow \nu_i q \bar{q}, \quad (25)$$

where $l = \{e, \mu\}$ and $q = \{u, d, s\}$. The corresponding leading order decay rate formulas are

$$\begin{aligned}
\Gamma(\nu_h \rightarrow l^- q_1 \bar{q}_2) &= |U_{lh}|^2 \frac{G_F^2}{64\pi^3} m_h^5 (|V_{ud}|^2 + |V_{us}|^2) \equiv |U_{lh}|^2 \Gamma^{(lX)}, \\
\Gamma(\nu_h \rightarrow \nu_i q \bar{q}) &= |\eta_i|^2 \frac{G_F^2}{3072\pi^3} m_h^5 [9 - 16 \sin^2 \theta_W (1 - \sin^2 \theta_W)] \equiv |\eta_i|^2 \Gamma^{(\nu X)}.
\end{aligned} \quad (26)$$

Here we neglected small $c\bar{d}, d\bar{c}$ contributions.

The following comment on the total NC decay rate $\Gamma_{\nu h}^{NC}$ is in order. Summation over all the NC channels gives

$$\begin{aligned}
\Gamma_{\nu h}^{NC} &= \Gamma_{\nu h}^\nu \sum_{i=1}^3 |\eta_i|^2 = \Gamma_{\nu h}^\nu \sum_{\alpha, \beta=e, \mu, \tau} U_{\alpha h} U_{\beta h}^* \sum_{i=1}^3 U_{\alpha i}^* U_{\beta i} = \\
&= \Gamma_{\nu h}^\nu \sum_{\alpha, \beta=e, \mu, \tau} U_{\alpha h} U_{\beta h}^* (\delta_{\alpha\beta} - U_{\alpha h}^* U_{\beta h}) \approx \Gamma_{\nu h}^\nu \sum_{\alpha=e, \mu, \tau} |U_{\alpha h}|^2.
\end{aligned} \quad (27)$$

Here we assumed that there are only four neutrino mass eigenstates $\nu_{1,2,3}, \nu_h$ and used unitarity of the mixing matrix $U_{\alpha\beta}$. At the last step we neglected the

subdominant term $\sim U_{\alpha h}^4$. Thus, both CC and NC decay rates are proportional to $\sim |U_{\alpha h}|^2$ and, as we mentioned at the beginning of this section, *a priori* are equally important.

Taking into account this fact we collect the partial decay rates into the total ν_h -decay width

$$\Gamma_{\nu h} = |U_{eh}|^2 \left(\Gamma_{\nu h}^{(e)} + \Gamma_{\nu h}^{(\nu)} \right) + |U_{\mu h}|^2 \left(\Gamma_{\nu h}^{(\mu)} + \Gamma_{\nu h}^{(\nu)} \right) + |U_{\tau h}|^2 \Gamma_{\nu h}^{(\nu)} \quad (28)$$

with

$$\Gamma_{\nu h}^{(l)} = \left(\Gamma_3^{(e\mu)} + \Gamma_3^{(ll)} \right) + \theta(m_{\eta'} - m_h) \sum_M \Gamma_2^{(lM)} + \theta(m_h - m_{\eta'}) \Gamma^{(lX)}, \quad (29)$$

$$\Gamma_{\nu h}^{(\nu)} = \left(\Gamma_3^{(e)} + \Gamma_3^{(\mu)} \right) + \theta(m_{\eta'} - m_h) \sum_{M^0} \Gamma_2^{(\nu M^0)} + \theta(m_h - m_{\eta'}) \Gamma^{(\nu X)} \quad (30)$$

where the summations run over $M = \pi^+, K^+, \rho^+, K^{*+}$ and $M^0 = \pi^0, \eta, \rho^0, \omega$.

If neutrinos ν_h are Majorana particles, i.e. $\nu_h \equiv \nu_h^c$, then both $\nu_h \rightarrow l^- X (\Delta L = 0)$ and $\nu_h \rightarrow l^+ X^c (\Delta L = 2)$ decay channels are open. This results in multiplication of the right hand side of Eq. (29) by factor 2.

5 Bounds on heavy sterile neutrino masses and mixings

Substituting the total ν_h -decay width from Eq. (28) into the resonant formula (17), we can derive, from the experimental bounds (15), constraints on the ν_h neutrino mass m_h and the mixing matrix elements $U_{\alpha h}$. In general these constraints represent a hardly readable 4-dimensional exclusion plot. However under certain simplifying assumptions one can infer more valuable information on the individual size of the mixing matrix elements. In this paper we are interested in the $U_{\tau h}$ matrix element which is not constrained in the literature in the ν_h mass range (16) (see [7]). Recently the NOMAD collaboration[10] obtained constraints for $m_h < 190$ MeV which overlap with a small part of this range. Therefore, it would be interesting to infer individual constraints on this mixing matrix element, at least roughly. A reasonable simplifying assumption would be to take $|U_{\tau h}| \sim |U_{\mu j}| \sim |U_{ej}|$. Then from the experimental bounds (15) we derive a 2-dimensional $m_h - |U_{\tau h}|^2$ exclusion plot given in Fig. 2 for the case of Dirac ν_h . Multiplying the Dirac case limiting curve in Fig. 2 by the factor 2 one obtains the exclusion plot for the case of Majorana ν_h . For comparison we also present in

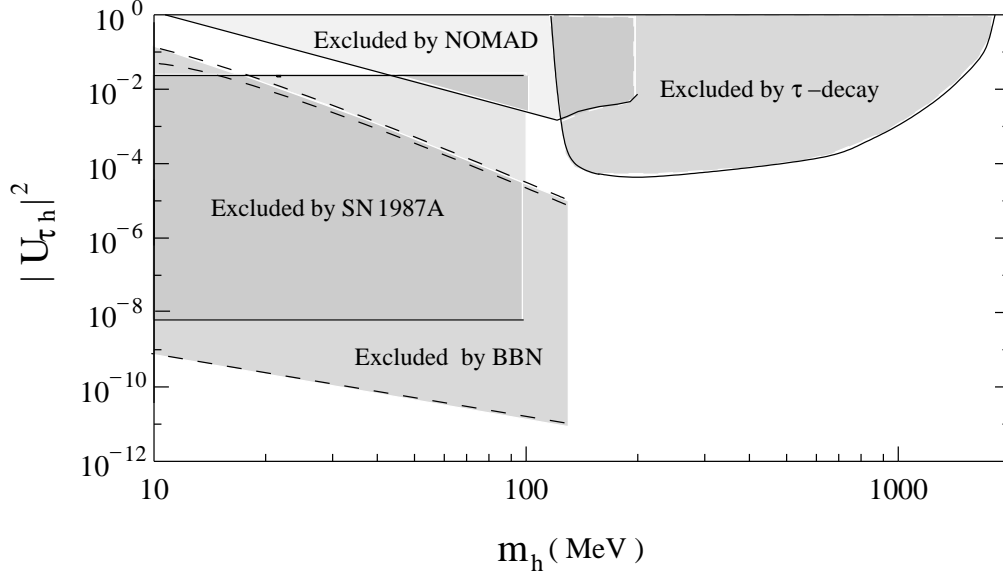


Figure 2: Exclusion plots in the plane $|U_{\tau h}|^2 - m_h$. Here $U_{\tau h}$ and m_h are the heavy neutrino ν_h mixing matrix element to ν_τ and its mass respectively. The shaded regions are excluded by the big bang nucleosynthesis, the duration of the SN 1987A neutrino burst [3], by the NOMAD collaboration [10] and by $\tau \rightarrow e^\pm(\mu^\pm)\pi^\mp\pi^-$ decay [present result].

Fig. 2 the bounds from big-bang nucleosynthesis, SN1987A [3] and the NOMAD bounds [10]. The cosmological bounds are shortly discussed in the next section.

Note that at certain points of the mass range (16) the limits in Fig. 2 are incompatible with our assumption that $|U_{\tau h}| \sim |U_{\mu j}| \sim |U_{ej}|$ since other experiments already established limits like $|U_{eh}|^2, |U_{\mu h}|^2 < 10^{-8}$ at $m_h = 350, 400$ MeV. Thus our limits around these points have to be treated with caution. Let us note that all known constraints on $|U_{lh}|$ [7] have been obtained under certain simplifying assumptions such as those that we used above.

However, there is a special case when limits on $|U_{\tau h}|$ can be extracted without any simplifying assumptions. This case is realized when there are experimental upper bounds on the rates of three processes involving $\tau\tau$, $\tau\mu$ and τe lepton pairs.

These could be, for instance, decays

$$B_c^+ \rightarrow \tau^+ \tau^+ \pi^-, \tau^+ \mu^+ \pi^-, \tau^+ e^+ \pi^-. \quad (31)$$

Intersection of their resonant regions is $1917 \text{ MeV} \leq m_h \leq 4620 \text{ MeV}$. In this domain the decay rates can be written schematically as

$$\begin{aligned} \Gamma_{\tau\tau} &= \frac{|U_{\tau h}|^4}{a|U_{eh}|^2 + b|U_{\mu h}|^2 + c|U_{\tau h}|^2}, & \Gamma_{\tau\mu} &= \frac{|U_{\tau h}|^2 |U_{\mu h}|^2}{a|U_{eh}|^2 + b|U_{\mu h}|^2 + c|U_{\tau h}|^2}, \\ \Gamma_{\tau e} &= \frac{|U_{\tau h}|^2 |U_{eh}|^2}{a|U_{eh}|^2 + b|U_{\mu h}|^2 + c|U_{\tau h}|^2}. \end{aligned} \quad (32)$$

From these Eqs. we find

$$|U_{\tau h}|^2 = a\Gamma_{\tau e} + b\Gamma_{\tau\mu} + c\Gamma_{\tau\tau}. \quad (33)$$

Notice that all the decay rates appear with positive degree. Only in such a case we can set upper bound on $|U_{\tau h}|^2$ having upper experimental bounds $\Gamma_{\tau l} \leq \text{Expl}$. This would not be the case, for instance, in the system $\Gamma_{\tau\mu}$, $\Gamma_{\tau e}$, Γ_{ee} .

At present the described approach can not be realized on practice due to absence of necessary experimental data. However the required upper limits on the semileptonic B_c -decays can be, probably, obtained in future experiments at the B-factories. Then, derivation of limits on $|U_{\tau h}|$ free of simplifying *ad hoc* assumptions will become possible.

6 Heavy sterile neutrinos in astrophysics and cosmology

It is well known that massive neutrinos may have important cosmological and astrophysical implications. They are expected to contribute to the mass density of the universe, participate in cosmic structure formation, big-bang nucleosynthesis, supernova explosions, imprint themselves in the cosmic microwave background etc. (for a review see, for instance, Ref. [14]). This implies certain constraints on the neutrino masses and mixings. Currently, for massive neutrinos in the mass region (16), the only available cosmological constraints arise from the mass density of the universe and cosmic structure formation.

The contribution of stable massive neutrinos to the mass density of the universe is described by the ‘‘Lee-Weinberg’’ $\Omega_\nu h^2 - m_\nu$ curve. From the requirement

that the universe is not “overclosed” this leads to the two well know solutions $m_\nu \leq 40\text{eV}$ and $m_\nu \gtrsim 10\text{GeV}$ which seem to exclude the domain Eq. (16). However for unstable neutrinos the situation is different. They may decay early to light particles and, therefore, their total energy can be significantly “redshifted” down to the “overclosing” limit. Constraints on the neutrino life times τ_{ν_j} and masses m_j in this scenario are found in Ref. [15]. In the mass region (16) we have an order of magnitude estimate

$$\tau_{\nu_j} < (\sim 10^{14})\text{sec} \quad \text{Mass Density limit} \quad (34)$$

Decaying massive neutrinos may also have specific impact on the cosmic structure formation introducing new stages in the evolution of the universe. After they decay into light relativistic particles the universe returns for a while from the matter to the radiation domination phase. This may change the resulting density fluctuation spectrum since the primordial fluctuations grow due to gravitation instability during the matter dominated stages. Comparison with observations leads to an upper bound on the neutrino life time [16]. In the mass region (16) one finds roughly

$$\tau_{\nu_j} < (\sim 10^7)\text{sec} \quad \text{Structure Formation limit} \quad (35)$$

On the other hand, from our formula (28) we estimate

$$\text{few sec} < \tau_{\nu_j}, \quad \text{Theoretical limit.} \quad (36)$$

Thus massive neutrinos with masses in the interval (16) are not yet excluded by the known cosmological constraints (34), (35).

Big-bang nucleosynthesis(BBN) and the SN 1987A neutrino signal may presumably lead to much more restrictive constraints [3]. The impact of heavy sterile neutrinos ν_h on the BBN is twofold: via their contribution to the cosmic energy density and via the secondary light neutrinos ν_e, ν_μ, ν_τ produced in ν_h -decays. The first issue results in faster expansion which enhances the neutron-to-proton ratio n/p . The secondary light neutrinos can produce also an opposite effect. The increased number of ν_e maintains neutron-proton thermal equilibrium longer diminishing the n/p -ratio at the freeze-out point. There are also effects related to the ν_e energy spectrum distortion and energy injection to the electromagnetic radiation from the e^+e^- pairs produced in ν_h -decays [3]. The observed duration of the neutrino signal from the SN 1987A stringently constraints the energy emitted from the core in the form of penetrating particles like neutrinos. The sterile

neutrinos could affect this energy-loss limits and, therefore, their mixings with the active neutrinos have to obey certain constraints.

Analysis of the BBN and SN 1987A constraints on ν_h parameters has been made in Ref. [3]. We present the corresponding exclusion plots in Fig. 2 for comparison with our limits. Unfortunately, the analysis of Ref. [3] is restricted to the mass range $m_h < 200$ MeV for the BBN and $m_h < 100$ MeV for the SN 1987A case. From the general appearance of the excluded bends one may expect that the extension of the SN 1987A bend can overlap with the region excluded by the τ -decay, producing very stringent combined constraint around $|U_{\tau h}|^2 < 10^{-8}$. Even stronger constraint would appear in the domain where the SN 1987A bend simultaneously overlaps with the BBN excluded bend. In this case a combined constraint would be, probably, more stringent than $|U_{\tau h}|^2 < 10^{-12}$.

7 Conclusion

We discussed a generic model with sterile neutrinos which can contain heavy ν_i mass eigenstates. If among them there are ν_h states with masses $140.5\text{MeV} \leq m_h \leq 1637\text{MeV}$ they can resonantly contribute to $\tau^- \rightarrow e^\pm(\mu^\pm)\pi^\mp\pi^-$ decays.

We derived decay rates for these processes valid both outside and inside of the resonant m_h region (16). In the latter case knowledge of the total ν_h -decay width $\Gamma_{\nu h}$ is required. We calculated the $\Gamma_{\nu h}$ in the whole resonant m_h region (16) taking into account both charged and neutral current decay channels.

The effect of resonant enhancement of the heavy sterile neutrino ν_h contribution allowed us to extract from the experimental upper bounds on the τ -decay rates rather stringent constraints on $\nu_\tau - \nu_h$ mixing matrix element $|U_{\tau h}|$ in the whole resonant region (16). The corresponding exclusion plot is presented in Fig. 2 together with the recent constraints from NOMAD collaboration [10] and cosmological constraints [3]. The τ -decay constraints were derived under certain simplifying assumptions about ν_h -mixing with the other ν_α flavors. These or similar assumptions are always required when ν_h -mixing with one selected ν_α flavor is extracted from those processes which have been studied in the literature. We propose the set of processes $B_c \rightarrow \tau\tau\pi, \tau\mu\pi, \tau e\pi, \mu\mu\pi, ee\pi$ which would allow one to avoid such *ad hoc* assumptions and extract individual constraints on $|U_{\tau h}|, |U_{\mu h}|, |U_{eh}|$. Finally, we discussed cosmological implications of the heavy sterile neutrinos ν_h and pointed out that extension of the known big bang nucleosynthesis and the SN 1987A constraints [3] to the resonant region (16) could significantly strengthen our constraints on $|U_{\tau h}|$ by combining them with these

cosmological constraints.

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